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Non-Fermi liquid from confinement in doped Mott insulators

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A phenomenological description for confinement of fractionalized excitations is proposed in the gauge theory approach for doped Mott insulators. Introducing the Polyakov-loop parameter into an SU(2) gauge theory for the t-J model, we show that electron excitations emerge below the so-called coherence temperature, resulting from confinement of spinons and holons via the formation of the Polyakov loop. Remarkably, such confined electrons turn out to exhibit non-Fermi liquid physics without quantum criticality, yielding the electric resistivity in quantitative agreement with experimental data. The Higgs phase is not allowed due to confinement, suggesting a possible novel mechanism of superconductivity in the strong coupling approach.

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Research on strongly correlated electrons gives rise to crisis in two cornerstones of modern theory of metals, Landau Fermi liquid theory and Landau-Ginzburg-Wilson framework for phase transitions [1]. Gauge theory formulation has been proposed to incorporate strong correlations, from which spin liquid physics, being described by fractionalized excitations [2], emerges as a main prediction. Although this strong coupling approach explains the anomalous charge and spin dynamics of high T_c cuprates at high temperatures, it fails to grasp why coherent electron excitations are observed at low temperatures [3]. This problem was often claimed to relate to confinement in the gauge theory, being regarded as an elusive one at the present technology.

Confinement is the most salient and difficult feature of quantum chromodynamics (QCD) [4]. Though lattice QCD simulations shed some light on it [5], it still defies any complete understanding. Recently, Fukushima proposed the Polyakov-loop extended Nambu-Jona-Lasinio (PNJL) model, where the Polyakov-loop parameter, measuring an effective potential for creation of static single quark [6], is introduced into the NJL model for the characterization of confinement [7]. This effective model has merits, in particular, in studying the quark matter at high temperatures, since both spontaneously broken chiral symmetry (SBχS) and confinement are described on an equal footing, being consistent with the lattice simulation [8].

In this Letter we introduce the PNJL scheme to the gauge theory approach for doped Mott insulators for the first time. Considering the Polyakov-loop parameter in an SU(2) slave-boson theory [3] of the t-J model, we show that electron excitations emerge below the so-called coherence temperature, being ascribed to confinement of spinons and holons via the formation of the Polyakov loop. Remarkably, such confined electrons turn out to exhibit non-Fermi liquid physics without quantum criticality, fitting experimental data for the electric resistivity quantitatively well. The Higgs phase given by the holon condensation is not allowed due to confinement, implying a novel mechanism of superconductivity, which is distinguished from the previous gauge theory approaches based on deconfinement.

We start from the SU(2) slave-boson representation of the t-J model [3]

$$
\begin{aligned}
Z &= \int D\psi_{i\alpha} Dh_i DU_{ij} Da^k_{i0} e^{-\int_0^\beta d\tau L}, \\
L &= \frac{1}{2}\sum_i \psi^\dagger_{i\alpha}(\partial_\tau - i a^k_{i0}\tau_k)\psi_{i\alpha} + J\sum_{\langle ij\rangle}(\psi^\dagger_{i\alpha}U_{ij}\psi_{j\alpha} \\
&+ H.c.) + \sum_i h^\dagger_i(\partial_\tau - \mu - i a^k_{i0}\tau_k)h_i \\
&+ t\sum_{\langle ij\rangle}(h^\dagger_i U_{ij} h_j + H.c.) + J\sum_{\langle ij\rangle}\mathbf{tr}[U^\dagger_{ij}U_{ij}], \qquad (1)
\end{aligned}
$$

where both spinon $\psi^\dagger_{i\sigma} = \begin{pmatrix} f^\dagger_{i\sigma} & \epsilon_{\sigma\sigma'} f_{i\sigma'} \end{pmatrix}$ and holon $h^\dagger_i = \begin{pmatrix} b^\dagger_{i1} & b^\dagger_{i2} \end{pmatrix}$ fields are given by doublets, carrying spin and charge quantum numbers of an electron, respectively. The order parameter matrix $U_{ij} = \begin{pmatrix} -\chi^\dagger_{ij} & \Delta_{ij} \\ \Delta^\dagger_{ij} & \chi_{ij} \end{pmatrix}$ comes from the standard decomposition for interactions, where χ_{ij} and Δ_{ij} are associated with particle-hole and particle-particle channels, respectively. Equation (1) should be regarded as one reformulation of the t-J model, decomposing an electron field into spinon and holon fields given by $c_{i\sigma} = \frac{1}{\sqrt{2}} h^\dagger_i \psi_{i\sigma}$, where the Gutzwiller projection is replaced with the exact integration of a^k_{i0}.

Employing the mean-field approximation for U_{ij} and a^k_{i0}, Wen and Lee [3] found the phase diagram of the effective theory represented by Eq. (1) in the (δ, T) plane with a fixed J/t, where δ denotes a hole concentration and T stands for temperature. The optimally hole-doped region at high temperatures is described by $U^{SM}_{ij} = -i\chi I$ and $\langle h_i\rangle = 0$ with $a^k_{i0} = 0$, called the strange metal (SM) phase, where spinons form a large Fermi surface, but only incoherent electron spectra are observed. The underdoped region at intermediate temperatures is characterized by $U^{SF}_{ij} = -\sqrt{\chi^2+\Delta^2}\tau_3 \exp[i(-1)^{i_x+i_y}\Phi\tau_3]$ with

$\Phi = \tan^{-1}\Big(\frac{\Delta}{\chi}\Big)$, $\langle h_i \rangle = 0$, and $a^k_{i0} = 0$, called the staggered flux (SF) phase, where spinons have Dirac spectrum due to the staggered internal flux Φ, but coherent electrons are not seen as the SM phase. Because the electron spectrum exhibits its spectral gap except for Dirac points, this SF state is identified with the so-called pseudogap phase in high T_c cuprates. Superconductivity results from condensation of holons $\langle b_{i1} \rangle \neq 0$ and $\langle b_{i2} \rangle = 0$ due to $ia^k_{i0} = \varphi \delta_{k3} \neq 0$ in the SF phase while the Fermi liquid state appears from the SM phase in the same way as the superconducting phase.

Low-energy physics and the stability of each phase should be investigated beyond the mean-field description, quantum fluctuations being introduced and an effective field theory being constructed. Considering quantum fluctuations $U^{SM}_{ij} = -i\chi e^{ia^k_{ij}\tau_k}$ in the SM phase, we can explain its low-energy physics by an SU(2) gauge theory

$$\begin{aligned}\mathcal{L}_{eff} &= \psi^\dagger_\alpha(\partial_\tau - ia^k_\tau\tau_k)\psi_\alpha + \frac{1}{2m_\psi}|(\partial_i - ia^k_i\tau_k)\psi_\alpha|^2 \\ &+ h^\dagger(\partial_\tau - \mu - ia^k_\tau\tau_k)h + \frac{1}{2m_h}|(\partial_i - ia^k_i\tau_k)h|^2 \\ &+ \frac{1}{4g^2}[\partial_\mu a^k_\nu - \partial_\nu a^k_\mu - g\epsilon_{klm}a^l_\mu a^m_\nu]^2, \end{aligned} \qquad (2)$$

where the time and space components of the SU(2) gauge fields arise from the Lagrange multiplier field and phase of the order parameter matrix, respectively. g stands for an effective coupling constant. In this effective field theory the spinons interact with the holons via SU(2) gauge fluctuations. Such gauge interactions have been proposed as the source for the anomalous transport in the SM phase [9]. However, it is not enough to treat gauge fluctuations perturbatively in order to simulate the Gutzwiller projection [10]. Moreover, such an approach based on deconfinement cannot recover the emergence of electron excitations at low temperatures without the Anderson-Higgs mechanism [3].

Defining the covariant derivative as

$$D_\mu = \partial_\mu - i\phi\tau_3\delta_{\mu\tau} - ia^k_\mu\tau_k,$$

where ϕ is the mean-field part of the gauge field associated with the Polyakov-loop parameter, and incorporating quantum fluctuations a^k_μ, we write down an effective PNJL model for the matter sector

$$\begin{aligned}\mathcal{L}^M_{PNJL} &= \psi^\dagger_\alpha(\partial_\tau - i\phi\tau_3)\psi_\alpha + \frac{1}{2m_\psi}|\partial_i\psi_\alpha|^2 \\ &+ h^\dagger(\partial_\tau - i\phi\tau_3 - \mu)h + \frac{1}{2m_h}|\partial_i h|^2 \\ &+ g_\psi\psi^\dagger_{\alpha n}\psi_{\alpha p}\psi^\dagger_{\beta p}\psi_{\beta n} + g_c\psi^\dagger_{\alpha n}\psi_{\alpha p}h^\dagger_p h_n, \end{aligned} \qquad (3)$$

where interactions between the spinons and holons are assumed to be local. This local approximation is well utilized in the QCD context, realizing SBχS successfully [8]. The local current-current interactions are expected to be irrelevant in the renormalization group sense, thus neglected for simplicity.

The spinon-exchange interaction, the first term of Eq. (3) in the last line, can be ignored in the SM phase while the electron resonance term will be allowed as quantum corrections, later. Then, we are led to the typical PNJL expression for the matter sector

$$\begin{aligned}F^{SM}_M[\Phi, \mu; \delta, T] &= -\frac{N_s}{\beta}\sum_k \ln\left(1 + 2\Phi e^{-\beta\frac{k^2}{2m_\psi}} + e^{-2\beta\frac{k^2}{2m_\psi}}\right) \\ &+ \frac{1}{\beta}\sum_q \ln\left(1 - 2\Phi e^{-\beta(\frac{q^2}{2m_h} - \mu)} + e^{-2\beta(\frac{q^2}{2m_h} - \mu)}\right) + \mu\delta, \end{aligned} \quad (4)$$

where μ denotes the chemical potential and $\Phi = \cos\beta\phi$ represents the Polyakov-loop parameter. Minimizing the free energy with respect to Φ, one always finds $\Phi = 1$, so that Eq. (4) is reduced to the deconfined theory. Matter fluctuations favor deconfinement as expected.

For the gauge sector, one can derive an effective theory of the Polyakov-loop order parameter from pure Yang-Mills theory, integrating over quantum fluctuations. Unfortunately, the gauge free energy from one-loop approximation always gives rise to $\Phi = 1$, i.e., deconfinement [11]. It is necessary to take quantum fluctuations into account in a non-perturbative way. Such a procedure is not known yet, and we construct an effective free energy as follows

$$F_G[\Phi; T] = A_4T^3\Big\{\frac{A_2T_0}{A_4}\Big(1 - \frac{T_0}{T}\Big)\Phi^2 - \frac{A_3}{A_4}\Phi^3 + \Phi^4\Big\}, \quad (5)$$

where the constants $A_{i=2,3,4}$ are positive definite, and T_0 is identified with the critical temperature for the confinement-deconfinement transition (CDT). Since the CDT is known as the first order from the lattice simulation [8], the cubic-power term with a negative constant is introduced such that $\Phi = 0$ in $T < T_0$ while $\Phi = 1$ in $T > T_0$, corresponding to the center symmetry (Z_2) breaking [7].

The resulting PNJL free energy is obtained as

$$F^{SM}_{PNJL}[\Phi, \mu; \delta, T] = F^{SM}_M[\Phi, \mu; \delta, T] + F_G[\Phi; T]. \qquad (6)$$

The CDT is driven by the gauge sector while the matter fluctuations turn the first order transition into the confinement-deconfinement crossover (CDC) because the Z_2 center symmetry is explicitly broken in the presence of matters, so that the Polyakov-loop does not become an order parameter in a rigorous sense [7]. One may regard this PNJL construction as our view point for the present problem according to experiments [12]. An important point is that confinement changes both spinon and holon spectra completely, allowing electron excitations, although feedback effects of matters to the Polyakov-loop parameter are not relevant.

Figure 1 shows the free energy as a function of the Polyakov-loop parameter for various temperatures, where $\Phi = 0$ in $T < T_{CD}$ (Black) and $\Phi = 1$ in $T > T_{CD}$ (Red) as it should be. The blue curve is drawn at $T = T_{CD}$. Here, T_{CD} is the CDC temperature in the presence of matters, smaller than T_0 because matters favor the deconfinement. An interesting point is that the chemical

potential of a negative value is much larger in the confinement phase than in the deconfinement phase, consistent with confinement. The inset displays the Polyakov-loop parameter that starts to appear around $T < T_0$.

An interesting result in the mean-field approach of the PNJL model is that condensation of bosons is not allowed, since the expression for the boson sector cannot reach the zero value because of $0 \le \Phi < 1$ except for $\Phi = 1$. In other words, Higgs phenomena are not compatible with confinement in this description, consistent with the previous field-theoretic result [13].

Since the Higgs phase is not allowed in the presence of the Polyakov-loop parameter, an immediate issue is how to describe the Fermi liquid phase, usually given by condensation of holons. We will examine the electron self-energy in the confinement phase, certainly recovering the Fermi-liquid self-energy proportional to ω^2 with frequency ω below a certain temperature associated with the holon chemical potential.

We repeat a similar study for the SF phase in which the spinon sector is described by the relativistic spectrum. The evolution of the Polyakov-loop parameter tends to be almost the same as the case of the SM phase. Note that the condensation of holons is also not allowed. Thus, the present framework presents a possible new mechanism of superconductivity in the presence of confinement instead of the holon condensation in the deconfinement phase.

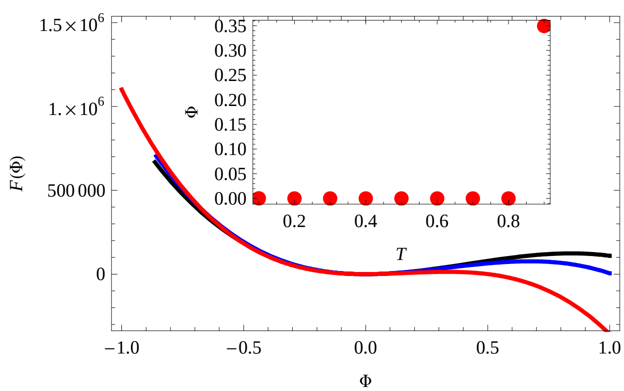

FIG. 1: (Color online) The effective PNJL free energy as a function of the Polyakov-loop parameter with $T < T_{CD}$ (Black), $T = T_{CD}$ (Blue), and $T > T_{CD}$ (Red). Inset: The Polyakov-loop parameter as a function of temperature scaled with T_0.

The central question of the present work is on the fate of the spinon and holon when the Polyakov-loop parameter vanishes. The spinon-holon coupling term in Eq. (3) can be expressed as follows

$$\mathcal{S}_{el} = \int_0^\beta d\tau \int d^2 r \left(\psi^\dagger_{\sigma n} h_n c_\sigma + c^\dagger_\sigma h^\dagger_p \psi_{\sigma p} - \frac{1}{g_c} c^\dagger_\sigma c_\sigma \right),$$

where σ and $n(p)$ represent spin and SU(2) indices, respectively. Since the Grassmann variable c_σ carries exactly the same quantum numbers with the electron, one may identify it as the Hubbard-Stratonovich field c_σ. The effective coupling constant g_c plays a role of the chemical potential for electrons. Note that the Fermi surface of the electrons differs from that of the spinons in principle.

One can introduce the quantum corrections self-consistently in the Luttinger-Ward functional approach [14] in which only planar diagrams are taken into account, ignoring vertex corrections [15]. We arrive at the self-consistent equations for self-energies

$$\begin{aligned}
\Sigma^c_{\sigma\sigma}(k, i\omega) &= -\frac{1}{\beta} \sum_{i\Omega} \sum_q G^h_{p'p}(q, i\Omega) G^\psi_{\sigma\sigma,pp'}(k-q, i\omega - i\Omega), \\
\Sigma^\psi_{\sigma\sigma,pp'}(k, i\omega) &= -\frac{1}{\beta} \sum_{i\Omega} \sum_q G^c_{\sigma\sigma}(k+q, i\omega + i\Omega) G^h_{p'p}(q, i\Omega), \\
\Sigma^h_{pp'}(q, i\Omega) &= \frac{1}{\beta} \sum_{i\omega} \sum_k G^c_{\sigma\sigma}(k+q, i\omega + i\Omega) G^\psi_{\sigma\sigma,pp'}(k, i\omega), \quad (7)
\end{aligned}$$

where the Green's functions for the electron, the spinon, and the holon are given as

$$\begin{aligned}
-G^{c-1}_{\sigma\sigma}(k, i\omega) &= \Sigma^c_{\sigma\sigma}(k, i\omega) - g_c^{-1}, \\
-G^{\psi-1}_{\sigma\sigma,pp'}(k, i\omega) &= -i(\omega + p\phi)\delta_{pp'} + \frac{k^2}{2m_\psi}\delta_{pp'} + \Sigma^\psi_{\sigma\sigma,pp'}(k, i\omega), \\
-G^{h-1}_{pp'}(q, i\Omega) &= [-i(\Omega + p\phi) - \mu]\delta_{pp'} + \frac{q^2}{2m_h}\delta_{pp'} \\
&\quad + \Sigma^h_{pp'}(q, i\Omega), \quad (8)
\end{aligned}$$

respectively. These equations were intensively discussed in the context of heavy fermions [15, 16] without confinement due to the Polyakov-loop parameter.

It is natural that the spectral function of the spinon should not be reduced to the delta function even if the self-energy correction is ignored owing to the presence of the background potential ϕ. That of the holon also features a broad structure even at the zero frequency because of the Polyakov-loop parameter. It indicates that both the spinon and holon are not well-defined excitations in the confinement phase. On the other hand, the electron as a spinon-holon composite exhibits a rather sharp peak, since the imaginary part of their self-energy vanishes in the zero frequency limit in spite of no pole structure in the Green's function.

The holon self-energy is found to be of the standard form in two dimensions

$$\begin{aligned}
&\Sigma^b_p(q, i\Omega) - \Sigma^b_p(q, 0) \\
&= -\frac{\rho_c}{i(\alpha - 1)} \Big\{ \tan^{-1}\Big(\frac{i\Omega + ip\phi - v^c_F q^* + v^c_F q}{-i\Omega - ip\phi + v^c_F q^* + v^c_F q} \Big) \\
&- \tan^{-1}\Big(\frac{i\Omega + ip\phi - \alpha v^c_F q^* + \alpha v^c_F q}{-i\Omega - ip\phi + \alpha v^c_F q^* + \alpha v^c_F q} \Big) \Big\} \quad (9)
\end{aligned}$$

except for $i\Omega \to i\Omega + ip\phi$. ρ_c is the density of states for the confined electron, and v^c_F stands for the corresponding Fermi velocity. α denotes the ratio of the electron band mass to the spinon one, given as almost unity. q^* designates the Fermi-momentum mismatch between the confined electron and spinon.

Inserting Eq. (9) into the electron self-energy equation, we can find its explicit form. An important energy scale

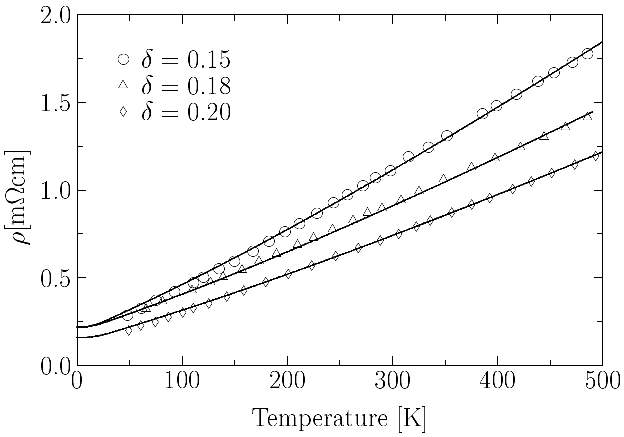

FIG. 2: (Color online) The electrical resistivity (Ref. [17]) with parameter $\mathcal{C}$ fitted.

is given by the holon chemical potential μ. In $T > |\mu|$ holon dynamics is described by the dynamical exponent $z = 3$, resulting from the Landau damping of the electron and spinon [15, 16]. The imaginary part of the self-energy turns out to be proportional to $T^{2/3}$, since the confined electrons are scattered with such $z = 3$ dissipative modes [15, 16]. On the other hand, the holon excitations have gaps in $T < |\mu|$, recovering the Fermi liquid. Thus, the Fermi liquid appears as the confinement phase rather than the Higgs in the PNJL approach.

We fit the resistivity data [17] for optimally doped cuprates. The relaxation time differs from the transport time, and the back scattering contribution is factored out by vertex corrections, corresponding to $T^{2/3}$ for two dimensional $z = 3$ fluctuations [18]. Then, the final expression can be written as

$$\rho_{el}(T) \;=\; \rho_0 + \mathcal{C}\Big(N_s \rho_c \frac{v_F^{c2}}{3}\Big)^{-1} T^{2/3} \Im\Sigma_c(T), \tag{10}$$

where ρ_0, $\mathcal{C}$, and N_s denote, respectively, the residual resistivity due to disorder, the strength for vertex corrections, and the spin degeneracy. ρ_0 and $\mathcal{C}$ are free parameters. Interestingly, the residual resistivity turns out to be almost constant for several hole doped samples near the optimal doping. Thus, we have practically only one free parameter, i.e. $\mathcal{C}$. As shown in Fig. 2, the results are in remarkable agreement with the data, which supports our confinement scenario.

In the present work, we have introduced an effect of confinement into the gauge theory approach for strongly correlated electrons, taking into account the Ployakov-loop parameter. We were able to identify the coherence temperature at which confinement of the spinon and holon, yielding electron excitations, emerges. Remarkably, such electron excitations are not fully coherent, which is consistent with the non-Fermi liquid physics observed in the optimally doped region. It was demonstrated explicitly by fitting the resistivity data (Fig. 2). A unique feature is that the Higgs mechanism does not arise in the presence of the Polyakov-loop parameter, which implies a possible novel mechanism of superconductivity beyond the existing gauge theoretical framework.

It will be of great interest to apply the PNJL scheme to the spin-liquid theory [2] and Kondo breakdown scenario [16] for heavy fermions. In the former the crossover behavior from spin 1 excitations to spin 1/2 can be investigated while this new mechanism for heavy fermions may occur in the latter.

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